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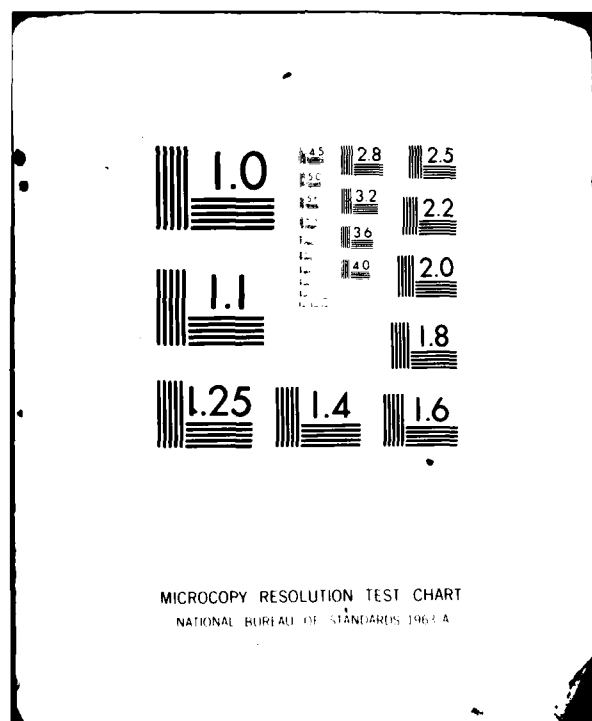
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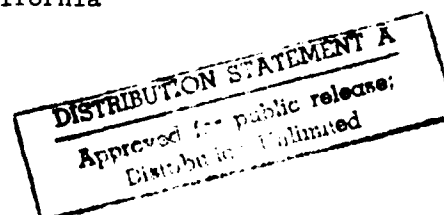
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ABSTRACT

This paper analyzes spatial models of electoral competitions with abstentions in which candidates have directional or local strategy sets. It includes, as a special case, situations in which incumbents must defend the status quo. The results derived here provide necessary and sufficient conditions for directional, convergent stationary and convergent local electoral equilibria for these spatial models. These conditions provide a method for finding such equilibria. They also provide existence results for directional, stationary and local electoral equilibria for societies with abstentions. The results on stationary and local electoral equilibria are obtained by analyzing cumulative plurality (or plurality potential) functions.



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EQUILIBRIUM POLICY PROPOSALS WITH ABSTENTIONS*

by

Peter Coughlin**

1. Introduction

Spatial analyses of economic policy formation in elections when voters may choose to abstain have been developed by Hinich and Ordeshook [1969] [1970], Hinich, Ledyard and Ordeshook [1972], Slutsky [1975], McKelvey [1975], Denzau and Kats [1977] and Hinich [1978]. One of the primary concerns of these investigations has been the determination of conditions under which there exist pure strategy equilibria for vote seeking candidates (and hence predictable outcomes). The only societies for which such equilibria have been shown to exist, thus far, have been ones in which the distribution of voters' ideal points is radially symmetric or special concavity conditions are satisfied. However, these special assumptions are highly restrictive and have additionally been criticized for being empirically ad hoc.^{1/} These (and related) analyses have also been criticized for assuming that candidates have perfect mobility (or global strategy sets).^{2/}

This paper, alternatively, considers the nature of electoral equilibria without including any special radial symmetry or concavity conditions. Additionally, it studies societies in which candidates have directional or local strategy sets^{3/} and includes in its analysis the important case in which incumbents must defend the status quo. The spatial voting model

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analyzed in this paper also includes probabilistic voter choices between candidates ^{4/} as well as between voting and abstaining. This analysis provides necessary and sufficient conditions for directional, stationary and local electoral equilibria. These conditions, in turn, provide general existence results for electoral equilibria when the society has an opportunity set which satisfies assumptions which are standard in microeconomics. All proofs are in the appendix.

2. Electoral Competitions with Abstentions^{5/}

The set of social alternatives is given by a non-empty, open, convex Euclidean policy space, $X \subset \mathbb{R}^n$. At any given time, the social opportunity set will be a feasible compact subset, $S \subset X$. In an electoral competition with perfect mobility, candidates compete for votes by proposing any of the feasible policies for the society. Their global or basic strategy set is therefore the set S .

$C = \{1,2\}$ will be an index set for the two candidates. Whenever we are considering a pair of proposals made by the candidates, they will always be ordered according to the candidates' indices. In particular, we will let $\psi_i \in S$ denote a basic strategy for candidate i . Then (ψ_1, ψ_2) will be a pair of policies proposed by the candidates.

Individual voters will be indexed by the elements, α , of a set $A \subset \mathbb{R}^n$. Their choice behavior will be summarized in (aggregate) probability functions,

$$(1) \quad p_{\alpha}^i: X \times X \rightarrow [0,1]$$

for $i = 0, 1, 2$ and $\alpha \in A$. $P_{\alpha}^i(\psi_1, \psi_2)$ with $i \in C$ is used to denote the probability that an individual who is randomly drawn from the citizens indexed by α will vote for candidate i when the pair (ψ_1, ψ_2) is proposed. $P_{\alpha}^0(\psi_1, \psi_2) = 1 - P_{\alpha}^1(\psi_1, \psi_2) - P_{\alpha}^2(\psi_1, \psi_2)$ is the probability that such an individual chooses to abstain.

We will additionally assume that (for $i = 1, 2$),

$$(2) \quad P_{\alpha}^i(\psi_1, \psi_2) = P_{\alpha}^i(G_{\alpha}(\psi_1), G_{\alpha}(\psi_2))$$

where $G_{\alpha}(\psi)$ is a scalar-valued function on X . This enables us to include both the utility-based probabilistic voting and abstentions of Hinich, Ledyard, Ordeshook, et al and the metric symmetry of McKelvey.^{6/} We will also take both the P_{α}^i and G_{α} to be twice continuously differentiable functions.^{7/} This follows from (but does not require) the assumptions about aggregate voting behavior in Hinich, Ledyard and Ordeshook (e.g. see [1972], pp. 147-148).

We will also assume that there is policy-related voting, by which we mean $P_{\alpha}^1(G_{\alpha}(x), G_{\alpha}(y)) = P_{\alpha}^2(G_{\alpha}(y), G_{\alpha}(x))$ for all $x, y \in X$. The policy related voting in McKelvey [1975] implies this assumption for the voting behavior which he has studied (when $G_{\alpha}(x) = U_{\alpha}(x)$). It also follows from the assumptions in Hinich, Ledyard and Ordeshook [1972], [1973] and Denzau and Kats [1977].

The population of individuals (and, hence, their aggregated choice probabilities) will be summarized by a probability measure space (A, A, μ) . For technical reasons (and with essentially no restriction) we assume that G_{α} , and P_{α}^i and their first and second partial derivatives are integrable with respect to this measure space. When the candidates have incomplete information about the distribution of individual characteristics in the population, this μ must be estimated. However, since candidates usually

have access to the same polls, past election data and other sources of information, we are implicitly assuming that in this case they have a common estimator^{8/} (as in Coughlin and Nitzan [1981]).

Finally, we will assume that candidates are interested in maximizing their expected pluralities.^{9/} $Pl_i(\psi_1, \psi_2)$ will be the notation used for the expected plurality^{10/} for candidate i at the pair of proposals $(\psi_1, \psi_2) \in X^2$.

3. Directional and Stationary Electoral Equilibria

In this section we will be concerned with situations in which candidates can (at most) marginally vary previously established positions. As is standard in microeconomic analyses, we will examine the consequences of candidates being concerned with the marginal changes in their respective expected pluralities which can result from their strategic choices.

At any basic strategy, $\psi_i^* \in S$, the directional strategy set for candidate i , $T(\psi_i^*)$, consists of all the feasible directions in S ^{11/} together with the zero vector in R^n (i.e. together with "no change"). We will use $u \in T(\psi_1^*)$ and $v \in T(\psi_2^*)$ to denote directions which may be selected by candidates 1 and 2, respectively.

The payoff function for candidate i , $i \in C$, (on the directional strategy sets) when the candidates are at the basic strategy pair $(\psi_1^*, \psi_2^*) \in S^2$ is given by the directional derivative (equation (15) in the appendix)

$$(3) \quad P_i(u, v) = D_{(u, v)} Pl_i(\psi_1, \psi_2) \quad \text{at} \quad (\psi_1, \psi_2) = (\psi_1^*, \psi_2^*)$$

for every $(u, v) \in T(\psi_1^*) \times T(\psi_2^*)$. This is simply the net effect on the candidates' plurality of the simultaneous variations in position by both the candidates.

Therefore, a pair of directions, $(u^*, v^*) \in T(\psi_1^*) \times T(\psi_2^*)$, is a directional electoral equilibrium (in pure strategies) at the basic pair of policies $(\psi_1^*, \psi_2^*) \in S^2$ if and only if

$$(4) \quad P_1(u, v^*) \leq P_1(u^*, v^*) \quad , \quad \forall u \in T(\psi_1^*) \quad , \quad \text{and}$$

$$P_2(u^*, v) \leq P_2(u^*, v^*) \quad , \quad \forall v \in T(\psi_2^*) \quad .$$

Theorem 1: $(u^*, v^*) \in T(\psi_1^*) \times T(\psi_2^*)$ is a directional electoral equilibrium at $(\psi_1^*, \psi_2^*) \in S^2$ if and only if u^* and v^* maximize the directional derivatives $D_u P_1(\psi_1, \psi_2^*)$ and $D_v P_2(\psi_1^*, \psi_2)$ at $\psi_1 = \psi_1^*$ and $\psi_2 = \psi_2^*$, respectively.

This implies a general existence result for such equilibria:

Corollary 1.1: There is a directional electoral equilibrium (in pure strategies) at every $(\psi_1^*, \psi_2^*) \in S^2$.

The strategic maneuvering of the candidates is in a state of rest if, and only if, both of them choose to remain at their current basic strategies. Therefore, to say that there is a stationary electoral equilibrium at the basic strategy pair $(\psi_1, \psi_2) \in S^2$ means that $(0, 0) \in R^{2n}$ is a directional electoral equilibrium at (ψ_1, ψ_2) .

The remainder of this section will be concerned with existence questions for stationary electoral equilibria. The analysis will answer these questions in the affirmative by showing, even more specifically, how to find convergent pairs of basic strategies (i.e., ones at which both candidates propose the same basic policies) where there are stationary electoral equilibria.

It should be observed that, thus far, the discussion has assumed that each of the candidates can vary his or her basic position in S in any feasible direction. However, quite often, an incumbent does not have this mobility and must, instead, defend the status quo.^{12/} Furthermore, challengers to a status quo or an incumbent might be restricted to feasible directions away from the status quo.^{13/}

Hence, we could alternatively study electoral games in which there is a fixed incumbent and a challenger whose objective is to maximize his expected plurality by appropriately varying his position in a feasible direction away from the status quo. However, we will show, this is equivalent to studying stationary electoral equilibria at convergent pairs of basic strategies. Consequently, this important case will be included in our analysis.

More formally, the possible payoffs available to the challenger at the status quo $\psi \in X$ are given by the directional derivative,

$$(5) \quad D_u P\ell_1(\psi_1, \psi) \text{ at } \psi_1 = \psi$$

for the $u \in T(\psi)$. An optimal strategy for a challenger is therefore any u^* which maximizes $D_u P\ell_1(\psi_1, \psi)$ at $\psi_1 = \psi$. Consequently, he'll be willing to not vary his position away from the status quo if and only if

$$(6) \quad D_u P\ell_1(\psi_1, \psi) \leq D_{\underline{0}} P\ell_1(\psi_1, \psi) \text{ at } \psi_1 = \psi$$

for every $u \in T(\psi)$.

Therefore we say that there is a stationary equilibrium at $\psi \in S$ when the incumbent must defend the status quo if and only if (6) is satisfied.

Theorem 2: There is a stationary electoral equilibrium at $(\psi, \psi) \in S^2$ if and only if there is a stationary equilibrium at ψ when the incumbent must defend the status quo.

We will now turn the discussion to some preliminary considerations which will enable us to define a third social choice mechanism. This third mechanism will be of special interest since it will provide us with necessary and sufficient conditions and general existence results for the stationary equilibria of Theorem 2.

Given an institutional setting in which only directional or infinitesimal changes are possible, a society may be concerned with the consequences of following different possible C^1 (or at least piecewise C^1) paths in its policy space. For instance, alternative paths could be compared on the cumulative total of the expected plurality (positive or negative) for changes along the paths.

To calculate this accumulated plurality, we will use the following notation for the gradient of marginal expected pluralities for changes from a status quo x ,

$$(7) \quad \nabla_1 P\ell_1(x, x) = \left[\frac{\partial P\ell_1(\psi_1, x)}{\partial \psi_{1h}} \right]_{\psi_1 = x}$$

where $h = 1, \dots, n$. Using this notation, the cumulative (marginal expected) plurality for the changes along a (piecewise C^1) path K from x_0 to x_1 is the line integral^{14/}

$$\begin{aligned}
 (8) \quad & \int_{x_0=K(a)}^{x_1=K(b)} \nabla_1 P \ell_1(K(\tau), K(\tau)) dK \\
 &= \int_a^b \nabla_1 P \ell_1(K(\tau), K(\tau)) \cdot K'(\tau) d\tau
 \end{aligned}$$

(where \cdot means inner product)..

It should be observed that this expected plurality would never be accumulated by a candidate who was competing against a rival (since both of them can make the same calculations and changes). However, this integral can be used as a measure of social approval or dissatisfaction with a path (i.e. with making all of the successive changes along K).

Since X is convex, there are many piecewise smooth paths from any $x_0 \in X$ to any $x_1 \in X$. Therefore, for any particular x_0 we can specify a function

$$(9) \quad K = K(x_0, x) = \phi(x; x_0)$$

which assigns a piecewise smooth curve from x_0 to x for each $x \in X$. For each specified x_0 and ϕ we then have a cumulative plurality function,

$$(10) \quad G(x) = G(x; x_0, \phi) = \int_{x_0=K(a)}^{x=K(b)} \nabla_1 P \ell_1(y, y) dK$$

defined on X (see (8)).

Any vector-valued function from R^n into R^n is called a vector field. Therefore, $F(x) = \nabla_1 P \ell_1(x, x)$ is clearly a vector field defined on X . Additionally, if $F(x)$ is a vector field and there exists a function

$f(x)$ for which $\nabla f(x) = F(x)$ for all $x \in X$, then F is called a gradient field and f is called a potential function for the gradient field.

For the class of electoral games being studied in this paper we have the following important result:

Theorem 3: $F(x) = \nabla_1 P\ell_1(x, x)$ is a gradient field.

One consequence of this theorem is

Corollary 3.1: The value of the line integral $\int_{x_0}^{x_1} \nabla_1 P\ell_1(y, y) dK$ is the same for every piecewise C^1 path K from x_0 to x_1 .

This says that, given $x_0 \in X$, the function $G(x) = G(x; x_0) = G(x; x_0, \phi)$ is independent of the function ϕ . We are therefore justified in calling $G(x; x_0)$ (see (10)) the cumulative plurality function at x_0 .

Theorem 3 also implies

Corollary 3.2: The cumulative plurality function at any given $x_0 \in X$ is a potential function for $F(x) = \nabla_1 P\ell_1(x, x)$.

The cumulative plurality function at a given $x_0 \in X$ will consequently also be referred to (interchangeably) as a plurality potential function.

This brings us to the third social choice mechanism which we will examine in this paper. Many procedures which have been suggested for making social choices involve the maximization of a social objective function (for instance, social welfare functions and Borda scores). When a society can choose among only feasible directions at a status quo it can, analogously, be concerned with maximizing the marginal change in its objective function. We will therefore refer to any $\psi \in X$ as a stationary outcome for the society's plurality potential function at $x_0 \in X$ if and only if

$$(11) \quad D_u G(x; x_0) \leq D_{\underline{0}} G(x; x_0) \quad \text{at } x = \psi$$

for every feasible direction, $u \in T(\psi)$.

This enables us to provide necessary and sufficient conditions for the stationary equilibria in Theorem 2 which specify their locations:

Theorem 4: There is a stationary electoral equilibrium at the basic strategy pair $(\psi, \psi) \in S^2$ if and only if ψ is a stationary outcome for the society's cumulative plurality function.

Our development of the cumulative plurality function also now provides us with the following general existence results:

Corollary 4.1: There exists a basic strategy pair $(\psi_1, \psi_2) \in S^2$ at which there is a stationary electoral equilibrium.

And even more specifically,

Corollary 4.2: There exists a convergent pair of basic strategies $(\psi, \psi) \in S^2$ at which there is a stationary electoral equilibrium, and

Corollary 4.3: There exists a status quo, $\psi \in S$, at which there is a stationary equilibrium when the incumbent must defend the status quo.

4. Local Electoral Equilibria

Stationary electoral equilibria which occur at critical points of a cumulative plurality function may have the undesirable property that each candidate is minimizing his objective function (while taking his rival's current policy position as given). Then, if the candidates obtain more local

information than just marginal pluralities, they will not remain at their positions. Alternatively, even if they know just their marginal pluralities but are perturbed to slightly different basic positions they may select directions for their positions which point away from (rather than back toward) the nearby stationary electoral equilibrium point. I.e., such equilibria can be locally unstable.

We therefore now consider existence questions for local equilibria. In particular, a local electoral equilibrium is a pair of basic strategies (ψ_1^*, ψ_2^*) such that

$$(12) \quad Pl_1(\psi_1, \psi_2^*) \leq Pl_1(\psi_1^*, \psi_2^*) \quad , \quad \text{and}$$

$$Pl_2(\psi_1^*, \psi_2) \leq Pl_2(\psi_1^*, \psi_2^*)$$

for every $\psi_1 \in N_{\epsilon_1}(\psi_1^*)$ and $\psi_2 \in N_{\epsilon_2}(\psi_2^*)$ for some $\epsilon_1, \epsilon_2 > 0$ (i.e., in some pair of ϵ -neighborhoods of ψ_1^* and ψ_2^*). Additionally, there is a local equilibrium at ψ when the incumbent must defend the status quo if and only if

$$Pl_1(\psi_1, \psi) \leq Pl_1(\psi, \psi)$$

for all $\psi_1 \in N_{\epsilon}(\psi)$ for some $\epsilon > 0$. These definitions are based on the concept of a local voting equilibrium in Kramer and Klevorick [1974].

We will study these local equilibria under assumptions on the social opportunity set, S , which are standard for such sets in microeconomic analyses. Specifically, we will assume that S is a compact subset of X which is defined by $m(<n)$ C^2 equations of the form

$$(13) \quad g_k(x) = 0 \quad (k = 1, \dots, m) \quad .$$

This means that the candidates and the society have Lagrangean maximization problems.

Additionally, in order to study this problem for C^2 payoff functions we will analyze those situations in which $G(x)$ (see (10)) and $P\ell_1(x,y)$ are both non-degenerate functions relative to the constraints given by (13).^{15/} It should be remarked that this imposes essentially no further restriction on the class of electoral competitions we are analyzing since, generically, every C^2 function is non-degenerate.^{16/}

Theorem 5: There is a local electoral equilibrium at the basic strategy pair $(\psi, \psi) \in S^2$ if and only if there is a local equilibrium at ψ when the incumbent must defend the status quo.

With this equivalence result in mind, we can also specify the locations of these local equilibria with a theorem which is analogous to Theorem 4:

Theorem 6: There is a local electoral equilibrium at the basic strategy pair $(\psi, \psi) \in S^2$ if and only if ψ is a local maximum of the society's cumulative plurality function at ψ .

It should be noted that this characterization of the convergent local electoral equilibria for the candidates is similar to the results of Arrow, Gould and Howe [1973] for local optimization problems. Here, however, we have found a constrained optimization problem which will solve our game's local saddle point problem, instead of the other way around.

Again, this gives us general existence results:

Corollary 6.1: There exists a basic strategy pair, $(\psi_1, \psi_2) \in S^2$, which is a local electoral equilibrium.

Furthermore,

Corollary 6.2: There exists a convergent pair of basic strategies, $(\psi, \psi) \in S^2$, which is a local electoral equilibrium, and

Corollary 6.3: There exists a social alternative in S at which there is a local equilibrium when the incumbent must defend the status quo.

5. Applications to Related Spatial Voting Models

This work has extended the earlier research of Hinich, Ledyard and Ordeshook [1972],[1973] (among others). In their papers they included special concavity assumptions. These additional assumptions are sufficient for the candidates' expected plurality functions to be strictly concave in their own strategies. It then followed that there was a unique global electoral equilibrium. What this does is to convert any stationary electoral equilibrium into a global electoral equilibrium. Their existence result, therefore, follows, alternatively, from Corollary 4.1 in this paper. The strictness of the concavity assures that the electoral equilibrium is unique. Convergence to the equilibrium then follows as in the original papers of Hinich, Ledyard and Ordeshook.

This work has also extended the research of Coughlin and Nitzan [1981] for electorates with probabilistic voting and no abstentions. This earlier paper derived a specific functional form for the P_{α}^i (see equation (1)) from assumptions in the Mathematical Psychology literature for binary choices. The resulting model is a special case for the analysis in this paper. Consequently, the existence theorems which were previously derived by Coughlin and Nitzan [1981] follow, alternatively, from Corollaries 5.1 and 6.1 in this paper.

6. Conclusion

This paper has analyzed electoral competitions with abstentions and probabilistic voting when candidates have directional, local or no mobility. It provides necessary and sufficient conditions for directional, convergent stationary and convergent local electoral equilibria. These conditions specify the locations of all of the convergent stationary and local electoral equilibria. They additionally provide general existence results for these equilibria.

Footnotes

- 1/ For example, see Slutsky [1975], Kramer [1977], and Rubenstein [1979].
- 2/ For example, see Matthews [1979] and Kramer and Klevorick [1974].
- 3/ Electoral competitions and majority rule with directional or local strategy sets or choice sets have previously been studied for societies with no abstentions in Plott [1967], Kramer and Klevorick [1974], Schofield [1978], Matthews [1979], Cohen and Matthews [1980], and Coughlin and Nitzan [1981].
- 4/ This incorporates the recent work of Hinich [1977], Kramer [1978] and Coughlin and Nitzan [1981] with the work on abstentions referred to in the first paragraph.
- 5/ The notations and assumptions in this paper are from the references cited in the first paragraph and in footnotes 1/-4/.
- 6/ For the utility-based probabilistic voting and abstentions of Hinich, Ledyard and Ordeshook [1972] and Denzau and Kats [1977], let $G_\alpha(x) = U_\alpha(x)$. For the metric symmetry of McKelvey [1975] (assumption 3.3), let $G_\alpha(x) = \|x - x_\alpha\|_A$, $P_\alpha^1(x,y) = W(G_\alpha(x), G_\alpha(y))$ and $P_\alpha^2(x,y) = W(G_\alpha(y), G_\alpha(x))$.
- 7/ It should be observed that the P_α^i will not be in C^2 when individuals with a common label have the same U_α utility function, everyone makes a deterministic choice of the candidate whose policy proposal has the greater utility for him and the candidates know the voters' behavior. However, it is easily satisfied in the aggregate when individuals choose probabilistically and/or candidates have smooth estimators for the voters' choice behavior.
- 8/ Of course, in directional and local electoral competitions the candidates will only obtain the portions of this estimator or of the functions which we derive from it that pertain to their possible strategies.
- 9/ When there is no chance element in voters' decisions, the maximization of expected plurality is simply the familiar maximization of plurality. When some or all of the voters' choices are probabilistic (or estimated as such), candidates could be concerned with their expected pluralities or their probabilities of winning. However, Hinich [1977] has shown that these two objectives are equivalent whenever there is a large population with probabilistic voting. Hence we consider only the first objective.

Footnotes Continued

- 10/ See Section I in the Appendix.
- 11/ The feasible directions in S at ψ_1^* are the n -vectors of unit length, w , for which there is some positive real number λ_w such that $\psi_1^* + \lambda \cdot w \in S$ for every $\lambda \in (0, \lambda_w)$.
- 12/ This is the structure in Kramer [1977], for instance.
- 13/ This is the structure in Plott [1967], Schofield [1978] and elsewhere.
- 14/ See Section IV in the Appendix.
- 15/ See Section V in the Appendix.
- 16/ E.g., see Hirsch [1976], Theorem 6.1.2.

Appendix

Section I. The expected pluralities for the candidates are given by:

$$(14) \quad P\ell_1(x,y) = \int_A (P_\alpha^1(x,y) - P_\alpha^2(x,y)) \cdot d\mu(\alpha) \quad , \quad \text{and}$$

$$P\ell_2(x,y) = \int_A (P_\alpha^2(x,y) - P_\alpha^1(x,y)) \cdot d\mu(\alpha) = -P\ell_1(x,y)$$

for every $(x,y) \in X^2$.

Section II. Proof of Theorem 1:

$$(15) \quad P_i(u,v) = D_{(u,v)} P\ell_i(\psi_1, \psi_2)$$

$$= \sum_{h=1}^n \frac{\partial P\ell_i(\psi_1, \psi_2)}{\partial \psi_{1h}} \cdot u_h + \sum_{h=1}^n \frac{\partial P\ell_i(\psi_1, \psi_2)}{\partial \psi_{2h}} \cdot v_h \quad .$$

Therefore, $P_i(u,v)$ is additively separable in u and v for both $i \in C$.

Hence any u^* and v^* which maximize

$$(16) \quad D_u P\ell_1(\psi_1, \psi_2^*) = \sum_{h=1}^n \frac{\partial P\ell_1(\psi_1, \psi_2^*)}{\partial \psi_{1h}} \cdot u_h$$

and

$$D_v P\ell_2(\psi_1^*, \psi_2) = \sum_{h=1}^n \frac{\partial P\ell_2(\psi_1^*, \psi_2)}{\partial \psi_{2h}} \cdot v_h$$

at $\psi_1 = \psi_1^*$ and $\psi_2 = \psi_2^*$, respectively, are dominant strategies.

Conversely if u^* (respectively, v^*) does not maximize $D_u Pl_1(\psi_1, \psi_2^*)$ (respectively, $D_v Pl_2(\psi_1^*, \psi_2)$) at $(\psi_1, \psi_2) = (\psi_1^*, \psi_2^*)$ then it is dominated by some other direction. Q.E.D.

Proof of Corollary 1.1:

First, since $D_u Pl_1(\psi_1, \psi_2)$ and $D_v Pl_2(\psi_1, \psi_2)$ are linear in u and v , they are also continuous in u and v , respectively. Secondly, since S is compact, $T(\psi_1^*)$ and $T(\psi_2^*)$ are compact. Therefore maxima exist. Q.E.D.

Section III. Proof of Theorem 2:

If there is a stationary electoral equilibrium at $(\psi, \psi) \in S^2$, then Theorem 1 directly implies that (6) is satisfied.

Conversely, if (6) is satisfied then we immediately know that $u^* = \underline{0}$ maximizes $D_u Pl_1(\psi_1, \psi)$ at $\psi_1 = \psi$. Therefore, we need only show that $v^* = \underline{0}$ is a dominant strategy for candidate 2.

By (14) and (2),

$$(17) \quad \begin{aligned} Pl_1(x, y) &= \int_A P_\alpha^1(G_\alpha(x), G_\alpha(y)) \cdot d\mu(\alpha) \\ &\quad - \int_A P_\alpha^2(G_\alpha(x), G_\alpha(y)) \cdot d\mu(\alpha) \end{aligned}$$

Therefore, since there is policy-related voting (p. 3),

$$(18) \quad \begin{aligned} Pl_1(x, y) &= \int_A P_\alpha^2(G_\alpha(y), G_\alpha(x)) \cdot d\mu(\alpha) \\ &\quad - \int_A P_\alpha^1(G_\alpha(y), G_\alpha(x)) \cdot d\mu(\alpha) \\ &= Pl_2(y, x) \end{aligned}$$

Consequently,

$$(19) \quad \frac{\partial \mathcal{P}\ell_2(\psi, \psi_2)}{\partial \psi_{2h}} = \frac{\partial \mathcal{P}\ell_1(\psi_1, \psi)}{\partial \psi_{1h}}$$

at $\psi_1 = \psi_2 = \psi$. Therefore,

$$(20) \quad \begin{aligned} D_v \mathcal{P}\ell_2(\psi, \psi_2) &= \sum_{h=1}^n \frac{\partial \mathcal{P}\ell_2(\psi, \psi_2)}{\partial \psi_{2h}} \cdot v_h \\ &= \sum_{h=1}^n \frac{\partial \mathcal{P}\ell_1(\psi_1, \psi)}{\partial \psi_{1h}} \cdot v_h \end{aligned}$$

at $\psi_1 = \psi_2 = \psi$.

Hence, since $u = \underline{0}$ maximizes $D_u \mathcal{P}\ell_1(\psi_1, \psi)$ at $\psi_1 = \psi$, it must also be true that $v = \underline{0}$ maximizes $D_v \mathcal{P}\ell_2(\psi, \psi_2)$ at $\psi_2 = \psi$. Q.E.D.

Section IV. A curve κ in R^n is a continuous function mapping a closed interval $[a, b]$ of real numbers into R^n . $\tau \in [a, b]$ is the parameter of the curve κ . The derivative of the curve $\kappa(\tau) = (\kappa_1(\tau), \dots, \kappa_n(\tau))$ with respect to the parameter τ is $\kappa'(\tau) = (d\kappa_1(\tau)/d\tau, \dots, d\kappa_n(\tau)/d\tau)$.

If the derivative $\kappa'(\tau)$ is continuous for all values of τ , then κ is called a smooth curve. If the interval $[a, b]$ may be partitioned into subintervals, $a = \tau_0 < \tau_1 < \dots < \tau_n = b$, such that κ is smooth on each of the subintervals, then κ is said to be piecewise smooth.

The line integral of a function h from R^n into R^n along a piecewise smooth curve κ is defined to be

$$\begin{aligned}
 (21) \quad \int_{\kappa}^{x_1=\kappa(b)} h(\kappa(\tau)) d\kappa &= \int_{x_0=\kappa(a)}^b h(\kappa(\tau)) \cdot \kappa'(\tau) d\tau \\
 &= \int_a^b h(\kappa(\tau)) \cdot \kappa'(\tau) d\tau
 \end{aligned}$$

(where \cdot is inner product). (See Curtis [1972] or Friedman [1971] for a thorough discussion of these integrals.)

Section V. Let $f(x)$ be a C^2 function on $X \subset \mathbb{R}^n$ and let $g_k(x) = 0$ ($k = 1, \dots, m < n$) be C^2 constraints on X which define a feasible set, $S \subset X$. Then a point x^* is called a critical point of f relative to these constraints if and only if it satisfies these constraints and has associated with it a Lagrange function

$$(22) \quad L = f + \lambda_1 g_1 + \dots + \lambda_m g_m$$

such that $\nabla F(x^*) = 0$.

f is said to be non-degenerate relative to these constraints if and only if the determinant of the Bordered Hessian

$$(23) \quad \left[\begin{array}{c|c} \frac{\partial L(x)}{\partial x_h \partial x_\ell} & \frac{\partial g_k(x)}{\partial x_h} \\ \hline \frac{\partial g_k(x)}{\partial x_\ell} & 0 \end{array} \right]$$

(where the dimensions of the sub-matrices are as follows:

$$\left[\frac{\partial L}{\partial x_h \partial x_\ell} \right] \text{ is } (n \times n) , \quad \left[\frac{\partial g_k(x)}{\partial x_h} \right] \text{ is } (n \times m) ,$$

$$\left[\frac{\partial g_k(x)}{\partial x_\ell} \right] \text{ is } (m \times n) \text{ and } [0] \text{ is } (m \times m))$$

is non-zero at every critical point of f relative to these constraints (see Hestenes [1975], p. 153).

Section VI. Proof of Theorem 3:

By Theorem 9.4 in Curtis [1972], if $F'(s)$ is a symmetric matrix for each $s \in X$ then it follows that $F(s) = \nabla_1 P\ell_1(s,s)$ is a gradient field. We therefore consider the entries in the matrix

$$(24) \quad F'(s) = \left[\frac{\partial^2 P\ell_1(s,s)}{\partial \psi_{1h} \partial \psi_{1k}} + \frac{\partial^2 P\ell_1(s,s)}{\partial \psi_{1h} \partial \psi_{2k}} \right]$$

with $h,k = 1, \dots, n$.

First,

$$(25) \quad \frac{\partial^2 P\ell_1(s,s)}{\partial \psi_{1h} \partial \psi_{1k}} = \frac{\partial^2 P\ell_1(s,s)}{\partial \psi_{1k} \partial \psi_{1h}}$$

by Young's Theorem (e.g., see Chiang [1974]) and Corollary 5.9 in Bartle [1965].

Secondly,

$$\begin{aligned}
 (26) \quad & \frac{\partial P \ell_1(x, y)}{\partial \psi_{1h}} \\
 &= \frac{\partial}{\partial \psi_{1h}} \left[\int_A P_{\alpha}^1(G_{\alpha}(x), G_{\alpha}(y)) \cdot d\mu(\alpha) - \int_A P_{\alpha}^2(G_{\alpha}(x), G_{\alpha}(y)) \cdot d\mu(\alpha) \right] \\
 &= \int_A \frac{\partial}{\partial \psi_{1h}} P_{\alpha}^1(G_{\alpha}(x), G_{\alpha}(y)) \cdot d\mu(\alpha) - \int_A \frac{\partial}{\partial \psi_{1h}} P_{\alpha}^2(G_{\alpha}(x), G_{\alpha}(y)) \cdot d\mu(\alpha) \\
 &= \int_A \frac{\partial P_{\alpha}^1(G_{\alpha}(x), G_{\alpha}(y))}{\partial G_{\alpha}(x)} \cdot \frac{\partial G_{\alpha}(x)}{\partial x_h} \cdot d\mu(\alpha) \\
 &\quad - \int_A \frac{\partial P_{\alpha}^2(G_{\alpha}(x), G_{\alpha}(y))}{\partial G_{\alpha}(x)} \cdot \frac{\partial G_{\alpha}(x)}{\partial x_h} \cdot d\mu(\alpha)
 \end{aligned}$$

by (14), (2), the Chain Rule (e.g., see Chiang [1974]) and Corollary 5.9 in Bartle [1965].

Therefore,

$$\begin{aligned}
 & \frac{\partial^2 P \ell_1(x, y)}{\partial \psi_{1h} \partial \psi_{2k}} \\
 &= \int_A \frac{\partial^2 P_{\alpha}^1(G_{\alpha}(x), G_{\alpha}(y))}{\partial G_{\alpha}(x) \partial G_{\alpha}(y)} \cdot \frac{\partial G_{\alpha}(y)}{\partial y_k} \cdot \frac{\partial G_{\alpha}(x)}{\partial x_h} \cdot d\mu(\alpha) \\
 &\quad - \int_A \frac{\partial^2 P_{\alpha}^2(G_{\alpha}(x), G_{\alpha}(y))}{\partial G_{\alpha}(x) \partial G_{\alpha}(y)} \cdot \frac{\partial G_{\alpha}(y)}{\partial y_k} \cdot \frac{\partial G_{\alpha}(x)}{\partial x_h} \cdot d\mu(\alpha)
 \end{aligned}$$

by Corollary 5.9 in Bartle [1975] and the Chain Rule.

Similarly,

$$\begin{aligned}
 (28) \quad & \frac{\partial^2 P\ell_1(x,y)}{\partial \psi_{1k} \partial \psi_{2h}} \\
 &= \int_A \frac{\partial^2 P_{\alpha}^1(G_{\alpha}(x), G_{\alpha}(y))}{\partial G_{\alpha}(x) \partial G_{\alpha}(y)} \cdot \frac{\partial G_{\alpha}(y)}{\partial y_h} \cdot \frac{\partial G_{\alpha}(x)}{\partial x_k} \cdot d\mu(\alpha) \\
 &\quad - \int_A \frac{\partial^2 P_{\alpha}^2(G_{\alpha}(x), G_{\alpha}(y))}{\partial G_{\alpha}(x) \partial G_{\alpha}(y)} \cdot \frac{\partial G_{\alpha}(y)}{\partial y_h} \cdot \frac{\partial G_{\alpha}(x)}{\partial x_k} \cdot d\mu(\alpha) .
 \end{aligned}$$

At $x = y = s$ we have

$$(29) \quad \left. \frac{\partial G_{\alpha}(y)}{\partial y_k} \cdot \frac{\partial G_{\alpha}(x)}{\partial x_h} \right|_{\substack{y=s \\ x=s}} = \left. \frac{\partial G_{\alpha}(x)}{\partial x_k} \cdot \frac{\partial G_{\alpha}(y)}{\partial y_h} \right|_{\substack{x=s \\ y=s}} .$$

Therefore,

$$(30) \quad \frac{\partial^2 P\ell_1(s,s)}{\partial \psi_{1h} \partial \psi_{2k}} = \frac{\partial^2 P\ell_1(s,s)}{\partial \psi_{1k} \partial \psi_{2h}} .$$

Finally, (25) and (30) imply that $\tilde{F}(s)$ is symmetric.

Therefore, $F(s) = \nabla_1 P\ell_1(s,s)$ is a gradient field on X . Q.E.D.

Proof of Corollary 3.1:

This follows directly from Theorem 3 (above) and Theorem 9.4.1 in Friedman [1971].

Proof of Corollary 3.2:

This follows directly from Theorem 3 (above) and Theorem 9.3 in Curtis [1972].

Section VI. Proof of Theorem 4:

By Corollary 3.2 and Theorem 9.3 in Curtis [1972],

$$(31) \quad \nabla G(y) = \nabla_1 P\ell_1(y, y)$$

at every $y \in S$. Recall that

$$(32) \quad D_u P\ell_1(y, y) = \nabla_1 P\ell_1(y, y) \cdot u$$

(where \cdot is inner product). Therefore,

$$\begin{aligned} (33) \quad D_u G(\psi) &= \nabla G(\psi) \cdot u \\ &= \nabla_1 P\ell_1(\psi, \psi) \cdot u \\ &= D_u P\ell_1(\psi_1, \psi) \Big|_{\psi_1 = \psi} \end{aligned}$$

Hence, Theorem 4 follows by an argument analogous to the proof of Theorem 2.

Proof of Corollary 4.1:

Follows directly from Corollary 4.2.

Proof of Corollary 4.2:

Since each $G(x)$ is continuously differentiable, $G(x)$ is a continuous function of x . Therefore, since S is compact, G must achieve a maximum

at some $\psi \in S$. Any such ψ must satisfy (10). Therefore, there must also be a stationary electoral equilibrium at (ψ, ψ) by Theorem 4.

Proof of Corollary 4.3:

By Corollary 4.2 and Theorem 2.

Section VII. Proof of Theorem 5:

This follows by a second-order argument analogous to the first-order argument in the proof of Theorem 2.

Proof of Theorem 6:

$$\begin{aligned}
 (34) \quad \left[\frac{\partial^2 G(y)}{\partial y_h \partial y_k} \right] &= \nabla \nabla G(y) \\
 &= \nabla (\nabla_1 P \ell_1(y, y)) \\
 &= \left[\frac{\partial^2 P \ell_1(y, y)}{\partial \psi_{1h} \partial \psi_{1k}} + \frac{\partial^2 P \ell_1(y, y)}{\partial \psi_{1h} \partial \psi_{2k}} \right].
 \end{aligned}$$

By the proof of Theorem 2, $P \ell_1(x, y) = -P \ell_1(y, x)$. Therefore,

$$(35) \quad \frac{\partial^2 P \ell_1(x, y)}{\partial \psi_{1h} \partial \psi_{2k}} = \frac{-\partial^2 P \ell_1(y, x)}{\partial \psi_{2h} \partial \psi_{1k}}.$$

But, by the proof of Theorem 3, when $x = y = \psi$ we have

$$(36) \quad \frac{\partial^2 P \ell_1(x, y)}{\partial \psi_{1h} \partial \psi_{2k}} = \frac{\partial^2 P \ell_1(x, y)}{\partial \psi_{2h} \partial \psi_{1k}}.$$

Therefore,

$$(37) \quad \frac{\partial^2 P\ell_1(\psi, \psi)}{\partial \psi_{1h} \partial \psi_{2k}} = 0$$

at every ψ for $h, k = 1, \dots, n$.

Hence,

$$(38) \quad \left[\frac{\partial^2 G(y)}{\partial y_h \partial y_k} \right] = \left[\frac{\partial^2 P\ell_1(y, y)}{\partial \psi_{1h} \partial \psi_{1k}} \right]$$

(with $h, k = 1, \dots, n$).

Now, suppose that there is a local electoral equilibrium at $\psi \in S$. Then, since $P\ell_1(\psi_1, \psi)$ is non-degenerate with respect to the constraints given by (13), there exist unique multipliers $\lambda_1, \dots, \lambda_m$ such that, if we set $L_1(x) = P\ell_1(x, \psi) + \lambda_1 g_1(x) + \dots + \lambda_m g_m(x)$, then $\nabla L_1(\psi) = 0$ and

$$(39) \quad L_1''(\psi) = \sum_{\ell=1}^n \sum_{h=1}^n \frac{\partial^2 L_1(\psi)}{\partial x_h \partial x_\ell} \cdot w_h \cdot w_\ell > 0$$

for every non-zero vector $w \in \mathbb{R}^n$ which satisfies the equation $\nabla g_k(\psi) \cdot w = 0$ ($k = 1, \dots, m$). (E.g., see the proof of Theorem 3.3.2 in Hestenes [1975]).

Now consider $G(x)$. Since $\nabla G(x) = \nabla P\ell_1(x, \psi)$ at $x = \psi$, we must have $L_2(x) = G(x) + \lambda_1 g_1(x) + \dots + \lambda_m g_m(x)$ for the same $\lambda_1, \dots, \lambda_m$ implies $\nabla L_2(x) = 0$. Additionally, (38) and (39) imply

$$(40) \quad L_2''(\psi) = \sum_{\ell=1}^n \sum_{h=1}^n \frac{\partial^2 L_2(\psi)}{\partial x_h \partial x_\ell} \cdot w_h \cdot w_\ell > 0$$

for every non-zero vector $w \in \mathbb{R}^n$ which satisfies $\nabla g_k(\psi) \cdot w = 0$ ($k = 1, \dots, m$). Therefore, since $G(x)$ is non-degenerate with respect to the constraints given by (15), ψ must be a strict local maximum of $L(x)$ (e.g., see Theorem 3.3.2 in Hestenes).

The converse follows similarly (as in Theorems 2 and 4). Q.E.D.

Proof of Corollary 6.1:

By Corollary 6.2.

Proof of Corollary 6.2:

By the proof of Corollary 4.2, $G(x)$ must achieve a local maximum at some $\psi \in S$. By Theorem 6, there is a local equilibrium whenever both candidates choose the strategy ψ (i.e., at $(\psi, \psi) \in S^2$).

Proof of Corollary 6.3:

By Corollary 6.2 and Theorem 2.

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